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Dynamic Effects of Member Failure on Response of Truss-Type Space Structures

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Besides the usual causes of structural failure such as material defects and fabrication and construction errors, damage to space structures can come from such adverse events as impact by foreign objects, docking, drastic temperature changes, and exposure to radiation and charged particles. This paper presents a methodology to determine effects of member failure on the dynamic response of a truss-type space structure. Emphasis has been given to effects of the dynamic nature of member failure on the structural response. Two types of member failure are considered: one, the sudden brittle-type damage or failure of a member, and the other, member snap or dynamic jump due to buckling. The study is specially directed toward potential progressive member failure in the structure. It includes the postbuckling regime of member behavior. A three-dimensional truss-type structure is analyzed. Results are presented to delineate dynamic effects of member failure on the overall structural response measured in terms of deformations and stresses.

Nomenclature

A	= member area of cross section
E	= modulus of elasticity of the structural material
$\{f_i\}$	= forces applied at the end joints of ruptured member i
I	= moment of inertia
$[K]$	= stiffness matrix of the intact structure
$[K']$	= stiffness matrix of the damaged structure
k	= a factor, $(N/EI)^{\frac{1}{2}}$
L	= length of a straight pin-ended member
M	= bending moment applied to a member
M_0	= limit moment in pure bending
$[M]$	= mass matrix of the structure
N	= member axial load: tension (positive) and compression (negative)
N_E	= Euler critical (buckling) load
N_0	= limit axial loading in pure tension and compression
N_Y	= axial yielding load
$\{P_0\}$	= magnitude of the static load applied at each node
$\{p\}$	= static joint load vector
t	= time
t_i	= time at which member i fails
t_r	= rise time for loads/forces
W_m	= value of w at the midlength point of the member
w	= lateral deflection of a member at a distance x from one end
x	= distance along the axis of the straight member
$\{y\}$	= structural-joint displacement vector
$\{\ddot{y}\}$	= structural-joint acceleration vector
$\{y_0\}$	= vector of initial displacements
$\{\ddot{y}_0\}$	= vector of initial accelerations
$\{y'\}$	= displacement vector of the damaged structure
$\{\ddot{y}'\}$	= acceleration vector of the damaged structure
Δ	= total member axial deflection
Δ_e	= uniform elastic axial deformation given by Hooke's law
Δ_g	= axial deformation due to the lateral deflection
Δ_p	= plastic axial deformation at the plastic hinge
Δ_t	= axial deformation due to plastic elongation, when $N = N_0$
ϵ	= axial strain
λ	= joint stiffness due to the rest of the structure
σ	= axial stress
θ	= slope next to the midlength point of the member
θ^*	= constant value of slope θ
d/dx	= differentiation with respect to variable x
d/dΔ	= differentiation with respect to axial displacement Δ

Introduction

BECAUSE of their light weight, high stiffness, and construction efficiency, truss-type structures are the preferred candidates for various large-scale construction projects in space. Structures in space are subjected to extreme environments such as radiation and extreme temperature variation. They have greater possibility of being struck by meteors and other orbiting objects. They are subjected to in-house vibration. Any of these causes may inflict physical damage on or may cause overstressing of a member or a portion of the structure. Although the member damage (local damage) affects a small part of a structure initially, it has potential for propagating to other parts and may ultimately cause total collapse of the structure. This type of failure has been termed "progressive collapse."

In practice, truss-type structures are designed to have many redundant members. Nevertheless, collapse of such structures is not uncommon and can have catastrophic consequences, as exemplified by numerous lattice roof collapses on earth.[1,2] The loss or failure of a member in a redundant trusslike structure causes force redistribution among the remaining members. The redistribution may cause these remaining members to carry greater loads, forcing them to fail by yielding or buckling. Analyses based on static load redistribution of terrestrial double-layer grid trusses have shown that the loss of only one critical member can lead to total collapse of the structure at full service loadings.[3] It would be expected that for orbital structures similar adverse behavior could occur.

In reality, the failure or rupture of members in a lattice truss is realized to be dynamic in nature.[3–6] It is therefore of practical importance that the contribution of such dynamic effects of member failure on the response of the structure be understood. Although several studies of truss-type structures considering static member failure have been reported in the literature, there are only few published studies dealing with effects of dynamic member failure on the response of the total structure.[7,8] Techniques have been developed to evaluate the structural integrity and detect damage of large orbiting structures using vibration measurements.[9] However, any technique

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of damage detection and health monitoring of the space structure should take account of possible member failure.

The objective of this study is to present a methodology to investigate the dynamic effects of the member failure on the response of a truss-type space structure. Two types of member failures are considered. One is the sudden (brittle-type) rupture of the member. Members may fail in this way for reasons such as material defects, fabrication or construction errors, impact, and accident. The other type of member failure considered is buckling of the member. A member under compression may snap after reaching critical load, and the structure may experience dynamic jump. The member will have reduced load carrying capacity in the postbuckling regime. Results of the structural response (e.g., displacements, member stresses, and frequencies) obtained from dynamic analyses of a three-dimensional truss-type space structure due to consecutive failure of two members are reported.

Analysis Methodology

As opposed to an externally applied dynamic loading, structural vibration in the problem considered is initiated by internal causes such as sudden rupture or loss of load-carrying capacity in one or more members.

Member Failure

In a truss-type structural system whose members are primarily carrying axial loads, member failure may take place by yielding (tension or compression) or by buckling of compression members. During dynamic loading, a tension member after yielding may take up additional load due to strain hardening,[4,10] whereas a buckled compressive member loses strength and sheds load onto other members.[11] Moreover, the critical buckling stress beyond which a member buckles is normally far smaller than the yield stress. In the present analysis, emphasis is therefore given to member failure under compression. Two major types of sudden member failures that can give dynamic effects in the structure are the brittle type and a member snap accompanied by a dynamic jump due to buckling.

Brittle-Type Member Failure

This type of failure occurs in the linear elastic region. When a member fails in this fashion it can be assumed that it no longer has any load-carrying capacity. For material that is not brittle, this type of failure can arise for reasons including material defects, fabrication or construction errors, impact, and accident.

Member Failure Due to Buckling

A compressive structural member buckles after reaching its critical stress and enters into postbuckling. The load-carrying capacity of a buckled member is substantially less than that of a prebuckled member. Under a constant applied load, the buckling of a member usually leads to inelastic postbuckling. The postbuckling behavior of a truss member is greatly affected by its slenderness ratio. A failure of a member in a redundant truss system has potential to cause stress reversal in other members. The exact course of stress reversal in a buckled member is difficult to model analytically. Not enough experimental results have been reported in this area.

Several analytical and experimental studies have been reported on the behavior of a structural member under the action of cyclic axial load applied quasistatically.[12–16] Typical behavior of a pin-ended member under a cyclic axial load is shown in Fig. 1. The curve of axial load N versus axial deflection Δ in this figure shows various stages of the member behavior: elastic prebuckling (OA), elastic buckling (AB), inelastic postbuckling (BC), elastic unloading and tensioning (CDE), elastic-plastic tensioning (EF), plastic tensioning (FG), and elastic unloading (GH).

The study reported by Nonaka[12] for a pin-ended brace under repeated axial loading gives closed-form relations for various stages involved during the member behavior. This method is used in the present study.

The axial deformation Δ, the relative displacement between the ends of the bar, is expresed as the sum of the four components

$$\Delta = \Delta_e + \Delta_g + \Delta_p + \Delta_t \tag{1}$$

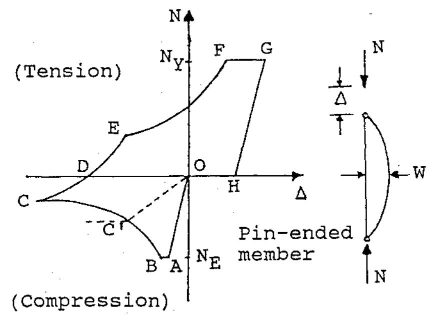


Fig. 1 Typical axial load–axial deformation relation of a pin-ended member.

Δ is taken positive when the distance between the ends of the bar increases.

1) The first component Δ_e of Eq. (1) is the uniform elastic deformation given by Hooke's law and equal to

$$\Delta_e = NL/AE \tag{2}$$

N, the axial load at the member end, is taken positive when tensile and negative when compressive (Appendix, Fig. A1).

2) The second component Δ_g is the deformation due to the change in geometry caused by lateral deflection w. This is given by

$$\Delta_g = -\frac{1}{2}\int_0^L \left(\frac{dw}{dx}\right)^2 dx \tag{3}$$

This component is determined by solving the basic differential equation for the lateral deflection of the member (Appendix, Fig. A1):

$$EI\frac{d^2w}{dx^2} - Nw = 0 \tag{4}$$

3) The third component Δ_p is the plastic axial deformation at the plastic hinge. It is calculated by the flow rule associated with a yield condition. Nonaka used the linear yield condition

$$\left|\frac{N}{N_0}\right| + \left|\frac{M}{M_0}\right| = 1 \tag{5}$$

Although the linear yield condition is derived for an ideal I section with indefinitely thin web and flanges, it is observed that this condition produces a steeper decrease in the member capacity in the postbuckling regime (thus leading to greater likelihood of dynamic jump) than a parabolic yield condition, which is a more appropriate assumption for a tubular section.[15] For the sake of presenting a methodology to analyze a structure undergoing the dynamic jump (or sudden reduction in the member capacity due to buckling/postbuckling), this simpler yield condition, which gives closed-form relations for the axial load-deformation characteristic, is adopted in the present study.

4) The last component Δ_t is the deformation due to plastic elongation, when $N = N_0$, distributed along the bar axis. It is controlled by the displacement constraints. Since the present study deals with the compression regions, this quantity does not enter into the analysis.

Governing expressions for the axial displacement components Δ_g and Δ_p are presented in the Appendix. Assumptions made in the method are as follows:

1) The bar can be considered as a one-dimensional continuum with uniform material and cross section along its length and is supported at its ends by pins.

2) The load is applied at the ends along the axis of original straight position. The load varies in a quasistatic manner. There are no transverse forces acting on the bar.

3) It has an elastic–perfectly-plastic characteristic. Shear effects are neglected.

4) Change in geometry is taken into account. However, the deflections are considered small. Change in length of the member is also considered to be negligible.

5) Plastic action takes place only at the middle of the member with the formation of the plastic hinge.

6) Under compression, the straight member buckles when the compression reaches Euler's elastic buckling load ($N_E = \pi^2 EI/L^2$).

If the load shedding of the compressive member after its initial buckling is sufficiently mild, there exists a stable situation in the postbuckling regime. However, when the load shedding is abrupt, there occurs an unstable situation.[11] A compressive member of a truss-type structure, therefore, may snap through, suffering a sudden drop in its load-carrying capacity, after it reaches critical buckling stress (portion BC in Fig. 1). The structure may hence experience dynamic jump. This phenomenon depends on the stiffness of the member and the resistance provided by the rest of the structure. The dynamic jump ensues if the negative stiffness of the member is greater than the stiffness λ provided by the rest of the structure.[4,17] Mathematically,

$$-\frac{dN}{d\Delta} > \lambda \tag{6}$$

Analytical Representation of Member Failure in a Structural System

Brittle-Type Failure

It is assumed that the dynamic response of a trusslike structure to sudden (brittle) failure of a member may be considered equivalent to its response to an abrupt drop in the member's load-carrying capacity. To simulate the brittle failure of a member analytically, the member capacity is first replaced by a pair of equivalent forces, equal and opposite, applied externally, one at each end joint of the member. The magnitude of this pair of forces must be such that when they and other existing loads are applied to the structure with the corresponding member damaged, the damaged structure will have the same response as that of its intact counterpart subjected to only the other existing loads. These additional joint forces are reduced to zero at the instant the member is considered failed. The response of the structure can be obtained by using established dynamic analysis techniques. The procedure outlined above for member failure is described in mathematical form below.

Intact structure. Figure 2a shows a cantilever truss under a uniformly distributed load of intensity p. The equation of motion for undamped vibration in matrix form is

$$[M]\{\ddot{y}\} + [k]\{y\} = \{p\} \tag{7}$$

subjected to the initial conditions

$$\{y\}_{t=0} = \{y_0\}, \qquad \{\dot{y}\}_{t=0} = \{\dot{y}_0\} \tag{8}$$

To facilitate the dynamic analysis, the static loads $\{p\}$ are applied in quasistatic fashion (Fig. 2d) at each node (joint) of the structure. The time distributions of the loads are given by

$$\{p\} = \begin{cases} \{P_0\}(t/t_r) & \text{for} \quad 0 \le t \le t_r \\ \{P_0\} & \text{for} \quad t > t_r \end{cases} \tag{9}$$

To obtain the static effect of the forces thus applied, the rise time t_r for the force-versus-time curve should be greater than approximately 3 times the natural period of the structure.[18,19] Under the applied loads, the structure deflects to position 1 from position 0 (Fig. 2c). At this position, members of the truss will acquire certain axial internal forces. The first member damage occurs at this position, initiating the force redistribution and hence the possible chain reaction of member failure.

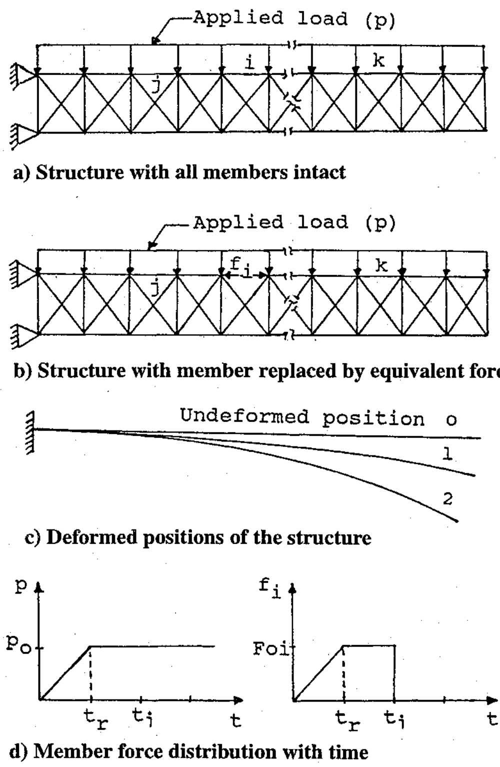
Damaged structure. Figure 2b shows the truss structure where the capacity of member i is replaced by an equivalent pair of forces $\{f_i\}$ applied at its end joints. The equation of motion for linear undamped vibration can be written as

$$[M]\{\ddot{y}'\} + [K']\{y'\} = \{p\} + \{f_i\} \tag{10}$$

Note that the mass matrix $[M]$ is left unaltered from Eq. (7), which indicates that the member remains as a part of the system without any load-carrying capacity. The distribution of $\{f_i\}$ with time is given in Fig. 2d:

$$\{f_i\} = \begin{cases} \{F_i\} \quad \text{for} \quad 0 \le t \le t_i \quad \text{(before member } i \text{ is damaged)} & \text{(11a)} \\ \{0\} \quad \text{for} \quad t > t_i \quad \text{(after member } i \text{ is damaged)} & \text{(11b)} \end{cases}$$

The value of $\{F_i\}$ in Eq. (11a) is to be computed, noting that the structural responses from Eq. (7) and Eq. (10) with $\{f_i\}$ given by Eq. (11a) are identical at all times before the member fails. That is, for $t \le t_i$, $\{y'\} = \{y\}$ and $\{\ddot{y}'\} = \{\ddot{y}\}$. Curve 2 in Fig. 2c shows a typical deflected position of the structure after member i fails at time t_i.



a) Structure with all members intact

b) Structure with member replaced by equivalent forces

c) Deformed positions of the structure

d) Member force distribution with time

Fig. 2 Physical member failure sequence in a truss structure.

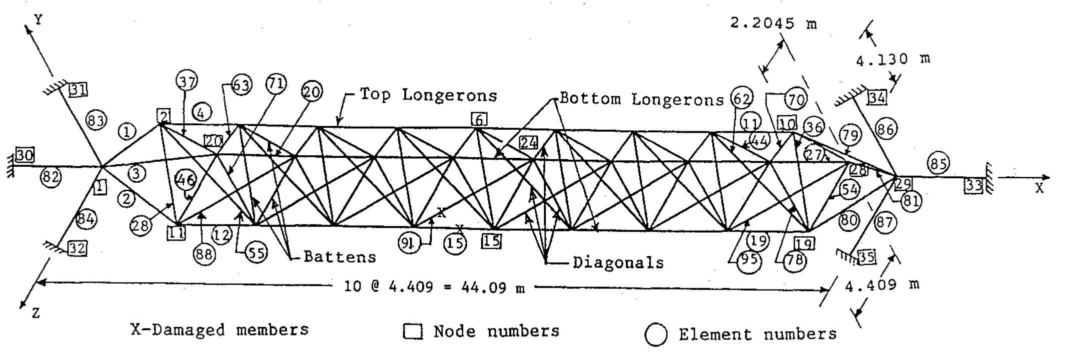


Fig. 3 Three-dimensional truss-type structure.

Table 1 Geometry, material properties and external loads

a) *Geometry*

Member cross section: circular tubes
Inner diam = 0.95 × (outer diam) for all members

Member	Outer diam, m	Area, 10^{-4} m^2	Element no.	Element type
Longerons	0.116	10.304	4–27	Truss
Battens	0.100	7.657	28–54	Truss
Diagonals	0.100	7.657	55–78, 88–95	Truss
End/Supports	0.164	2.061	1–3, 79–87	Beam

b) *Material properties*

Material: aluminum
Density $\rho = 2710$ kg/m^3
Yield stress $\sigma_y = 95$ MPa
Modulus of elasticity $E = 72.0$ GPa

c) *External loads*

Load, kN	Nodes	Dir.
0.5	4, 13, 22	$-X$
0.5	8, 17, 26	$+X$
60.0	14	$+X$
20.0	14	$-Z$
6.5	15	$-Z$
3.0	20–28	$+Z$

Buckling and Postbuckling Failure

For a member that is under compression and goes into buckling and postbuckling, the axial force–axial deformation behavior is obtained using the method outlined previously in the subsection Member Failure Due to Buckling. Once it is determined that the member goes to snap-through and dynamic jump [Eq. (6)] after initial buckling and enters into the postbuckling regime, the member material behavior is assumed to change suddenly to inelastic, especially the midlength portion where the plastic hinge is considered to form. At the same time a sudden decrease in member capacity occurs. Therefore, the midlength portion of the damaged (postbuckled) member is considered to have elastoplastic characteristic with reduced modulus of elasticity and yield stress corresponding to line OC' in Fig. 1. The rest of the damaged member is considered to be elastic but with reduced modulus.

The actual unloading from any point in the inelastic postbuckling region (BC) takes place along a curved path similar to CD. However, to locate point C' (and hence line OC') in the corresponding stress-strain behavior curve of the member for the elastoplastic analysis proposed here, two points are considered: 1) the extent of the decrease in the member capacity in the postbuckling regime, and 2) the fact that in the normal elastoplastic analysis the unloading from the plastic deformation region is assumed to take place parallel to the initial elastic (virgin) stress-strain path. Therefore, point C' is chosen such that line OC' is approximately parallel to line CD. The sudden process of motion (sudden drop in member capacity) from point B to C' is allowed for by providing the response of the structure corresponding to point B as initial conditions for the elastoplastic damaged-member structural model. The subsequent dynamic response of the structure is then obtained.

In reality, the structural vibration will ultimately die out because of damping in the structural material and/or joints. The present study aims at determining only the maximum transient response and therefore deals with the undamped situation.

Application and Results

Structural Model

The response and behavior of a three-dimensional, 10-bay, 44.09-m-long aluminum truss-type structure during member failure has been studied using the methodology outlined above. Figure 3 and Table 1 show the structure and its dimensions and material properties. The structure is supported at the two ends by beam members. To provide initial member stresses, the structure is subjected to static loads, whose magnitudes and directions are as indicated in Table 1. The loads are applied quasistatically to facilitate subsequent dynamic analysis.

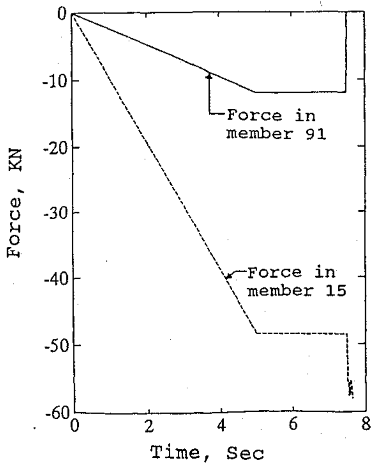


Fig. 4 Forces in members 15 and 91.

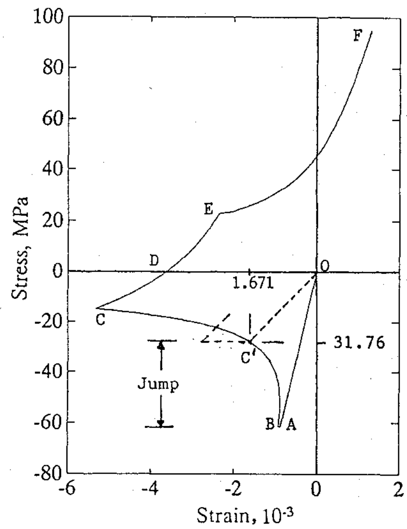


Fig. 5 Axial-stress–axial-strain relation for member 15.

Forces to replace the capacity of the member, which is determined to fail in brittle fashion, were computed from the dynamic analysis of the intact and damaged structures. Dynamic analyses to determine the displacements, stresses, forces, frequencies, and mode shapes of the structure were performed with the help of the COSMOS finite-element software package for a Sun computer system developed by the Structural Research and Analysis Corporation of Santa Monica, California. The nonlinear module with small deformation and elastoplastic option, where necessary, was used. As the updating of geometry is needed just before the failure of a member in the method presented, a step-by-step solution technique used in the nonlinear problem is essential. The Newmark-Beta direct time-integration method was used for solution. The Newton-Raphson numerical solution algorithm was utilized. The time step size chosen for integration was 0.02 s.

Member Failure Sequence and Representation

The structure is damaged first by suddenly decreasing the load-carrying capacity of member 91 to zero. This member is assumed to fail below its critical and yield stresses in a brittle fashion. This may be the case when the member has material defects, fabrication, or construction errors. Under the applied static load the force in member 91 is found to be approximately -11.9 kN. The member is assumed to fail at time 7.5 s. The sudden (brittle) failure of the member is simulated by replacing the member with the corresponding member force applied slowly in quasistatic manner from zero to its maximum value, and then dropping it to zero at time 7.5 s

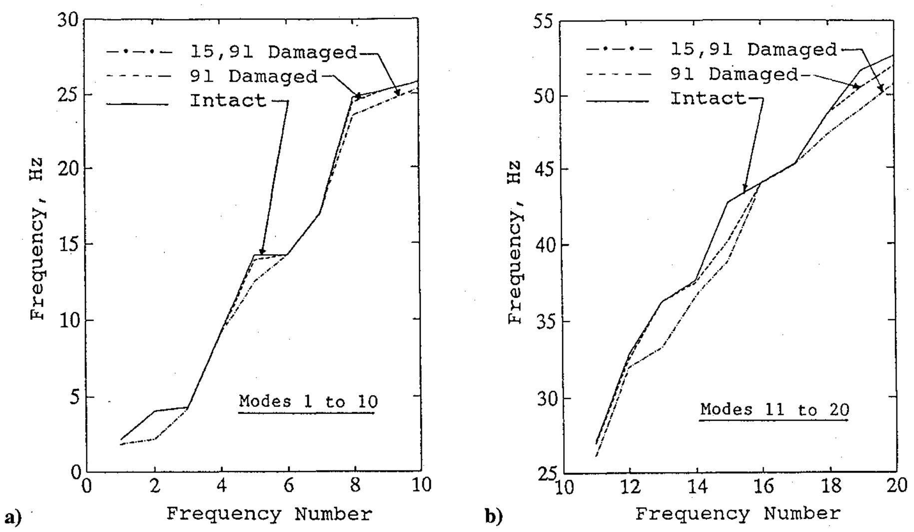


Fig. 6 Natural frequencies of the truss structure.

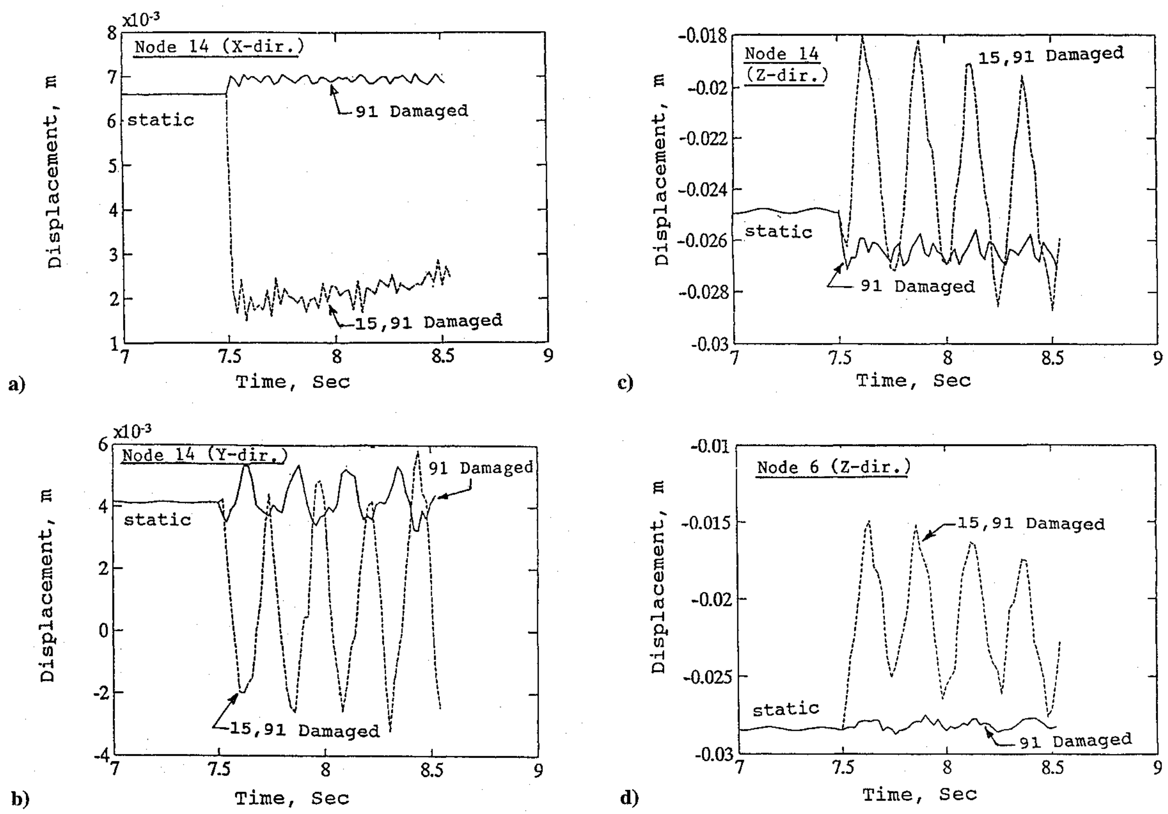


Fig. 7 Time variations of selected joint displacements.

(Fig. 4). The structure undergoes vibration due to the sudden failure of the member.

Member 15 is found to fail next by exceeding its Euler buckling load. Figure 4 shows the force in the member until it fails at time 7.64 s. The curves of axial load vs axial displacement (N-Δ) and average axial stress vs axial strain (σ-ϵ) for member 15 were obtained using the relations outlined briefly in the previous section and the Appendix. The stress-strain curve is shown in Fig. 5. The critical buckling stress of the member is computed to be equal to -58.489 MPa. It has a horizontal plateau AB (elastic buckling). Then it goes into inelastic postbuckling beyond point B. The maximum negative slope of this curve (negative member stiffness) is approximately 0.247×10^5 kN/m. The resistance (stiffness) from the rest of the structure at the joints where member 15 is connected is determined to be equal to 0.471×10^3 kN/m. The stress in member 15 after the failure of member 91 reaches a value close to its critical level at time 7.64 s. It is assumed that the member buckles at this time, subsequently snaps into the postbuckling regime, and experiences a sudden drop in its load-carrying capacity.

As the negative stiffness of member 15 is greater that the resistance provided by the rest of the structure, dynamic jump takes place. The member can carry only reduced load. Thus, dynamic force redistribution takes place in the structure. The member is now modeled with five intermediate finite elements. All elements in this member are assumed to have reduced modulus of elasticity equal to 19 GPa (shown by the dashed line in Fig. 5). The middle element is modeled to have elastoplastic material behavior, reflecting plastic-hinge formation, with reduced yield stress equal to 31.7 MPa (point C' in Fig. 5). The unloading (elastic recovery) from the postbuckling regime takes place along a line parallel to OC'. To get the dynamic

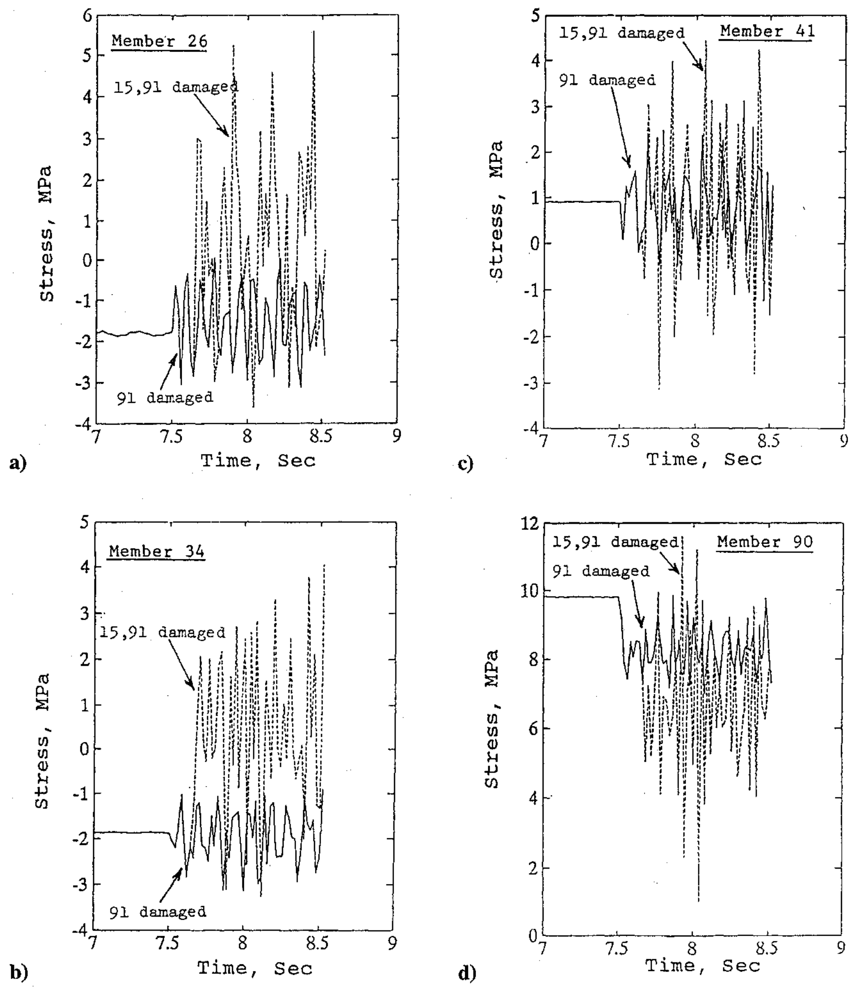


Fig. 8 Time variations of selected member axial stresses.

effects of member 15 failure, the damaged structure (i.e., the structure with member 91 having no load-carrying capacity and member 15 having reduced capacity) is subjected to initial conditions (joint initial displacements, velocities, and accelerations) as obtained at time 7.64 s from the structure with only member 91 damaged. The free-vibration analysis of the structure is performed for the subsequent time.

Structural Response

Static and dynamic analysis results were obtained. Some sample results are presented in Figs. 6–8. Figures 6a and 6b show the first 20 natural frequencies for these structures. A total of 87 frequencies were obtained. The frequencies range from 1.92 Hz (lowest when both the members are damaged) to a maximum of 240.76 Hz (intact structure). Figures 7a–7c show displacements with time for node 14 in the X, Y, and Z directions, respectively. Figure 7d shows the time variation of node 6 in the z direction. The stresses in members 26, 34, 41, and 90 are shown in Figs. 8a–8d, respectively.

Conclusions

An analysis technique has been outlined to determine dynamic effects of member damage on the behavior of a truss structure. Emphasis has been given to the dynamic nature of member failure. The method was employed for a three-dimensional truss-type end-supported structure subjected to member failure of brittle type and to snap-through and dynamic jump due to buckling. Some results have been presented delineating the dynamic effects of the member failure on the structure's displacements and member stresses. It must be pointed out, however, that the methodology used is simple and approximate. In-depth analysis taking account of the actual path of member behavior during its failure is needed for better understanding of the structural response during the member failure process. Of interest will also be the study of the effects of rate of loading due to member rupture.

Appendix: Axial Contributions from the Lateral Displacement and Plastic Action

This Appendix presents the governing closed-form expressions for the axial deformation components, Δ_g and Δ_p, of a pin-ended structural member that is subjected to axial loads at its ends.[12,15] Δ_g is the axial displacement contribution due to the lateral displacement, and Δ_p that due to plastic action at the middle of the pin-ended member (Fig. A1).

The solution to the basic differential equation (4) for the lateral deflection w of the pin-ended structural member is of the form

$$w = A_1 \sinh kx + B_1 \cosh kx \tag{A1}$$

where A_1 and B_1 are constants and $k = (N/EI)^{1/2}$. The end condition $w = 0$ at $x = 0$ gives $B_1 = 0$. Thus, Eq. (A1) becomes

$$w = A_1 \sinh kx \tag{A2}$$

1) When plastic action takes place (i.e., plastic-hinge formation at the midlength point of the bar), the constant A_1 in Eq. (A2) can be determined from the boundary condition: at $x = L/2$, $w = W_m$. With this condition the equation of the deflected shape of the member becomes

$$w = \frac{W_m}{\sinh(kL/2)} \sinh kx \tag{A3}$$

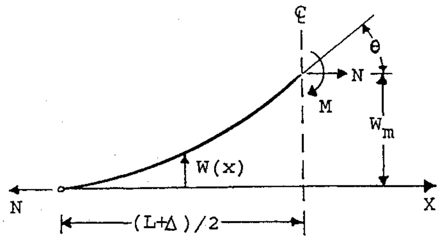


Fig. A1 Free-body diagram of a pin-ended bar under axial loads.

The slope θ next to the midlength point of the bar can be determined as

$$\theta = \left(\frac{dw}{dx}\right)_{x=L/2} = \frac{kW_m}{\tanh(kL/2)} \tag{A4}$$

Here W_m can be computed in terms of the axial force N with the help of the equilibrium equation of the half bar (Fig. A1),

$$M + NW_m = 0 \tag{A5}$$

and the moment–axial-force interaction relation given by Eq. (5). Substitution of Eq. (A3) into Eq. (3) and evaluation of the indicated integration yield

$$\Delta_g = -L\left(\frac{kW_m}{2\sinh(kL/2)}\right)^2\left(1 + \frac{\sinh kL}{kL}\right) \tag{A6}$$

It must be noted that the analytical derivation presented here is based on the tensile axial force being positive. When the axial force is compressive, n is negative; then W_m should be replaced by its absolute value, and the hyperbolic functions by the corresponding trigonometric functions. For instance, Δ_g at point B (Fig. 1), where the plastic hinge initiates in compression, can be obtained from Eq. (A6) with $N = N_E$ $(= \pi^2 EI/L^2)$ and $\sinh kx$ replaced by $\sin kx$ as

$$\Delta_g = -\frac{\pi^2 W_m^2}{4L}. \tag{A7}$$

2) For the elastic deformation stages following the plastic action (e.g., the elastic recovery portion CD and elastic tensioning portion DE in Fig. 1), the constant A_1 in Eq. (A2) is to be determined from the boundary condition that the slope θ next to the midlength point of the bar remains constant and is equal to that at the instant the elastic recovery begins (e.g., point C for region CDE):

$$\left(\frac{dw}{dx}\right)_{x=L/2} = \theta^*, \qquad \text{a constant} \tag{A8}$$

θ^* is computed from Eq. (A4) for the corresponding value of N. Using the boundary condition given by Eq. (A8) in Eq. (A2), the deflected shape is given by

$$w = \frac{\theta^* \sinh kx}{k \cosh(kL/2)} \tag{A9}$$

Substitution of Eq. (A9) into Eq. (3) gives

$$\Delta_g = -L\left(\frac{\theta^*}{2\cosh(kL/2)}\right)^2\left(1 + \frac{\sinh kL}{kL}\right) \tag{A10}$$

The third component Δ_p, plastic axial deformation at the plastic hinge, is calculated by the flow rule associated with a yield condition. In mathematical terms it can be expressed as[12,15]

$$\frac{d\Delta_p}{d\theta} = -2\frac{dM}{dN} \tag{A11}$$

For the linear moment–axial-load yield condition given by Eq. (5), this leads to

$$\Delta_p = -\int\left(\frac{2M_0}{N_0}\right)d\theta \tag{A12}$$

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